Multiprocessor task problems with parallel processors

- **maximal polynomially solvable:**

$P\|outtree; r_i; p_i = p\|C_{max}$	Brucker et al. (1977) [5]
$P\|tree; p_i = p\|C_{max}$	Hu (1961) [19], Davida & Linton (1976) [12]
$P2\|prec; p_i = p; size_i\|C_{max}$	Lloyd (1981) [25]
$Pm\|r_i; p_i = p; size_i\|C_{max}$	Baptiste (2003B) [1]
$P\|chains; r_i; p_i = 1\|L_{max}$	Dror et al. (1998) [13], Baptiste et al. (2004) [2]
$P\|intree; p_i = p\|L_{max}$	Brucker et al. (1977) [5], Monma (1982) [26]
$P2\|prec; r_i; p_i = 1\|L_{max}$	Garey & Johnson (1977) [17]
$P\|\|\sum C_i$	Bruno et al. (1974) [10]
$P\|outtree; p_i = p\|\sum C_i$	Hu (1961) [19]
$P\|outtree; r_i; p_i = 1\|\sum C_i$	Brucker et al. (2002) [6]
$P2\|prec; p_i = p\|\sum C_i$	Coffman & Graham (1972) [11]
$Pm\|intree; p_i = p\|\sum C_i$	Baptiste et al. (2004) [2]
$Pm\|r_i; p_i = p; size_i\|\sum C_i$	Baptiste (2003B) [1]
$P\|r_i; p_i = p\|\sum w_i C_i$	Brucker & Kravchenko (2008) [9]
$Pm\|p_i = p; size_i\|\sum w_i C_i$	Drozdowski & Dell' Olmo (2000) [14]
$P2\|r_i; p_i = 1; size_i\|L_{max}$	Baptiste & Schieber (2003) [3]
$Pm\|r_i; p_i = p\|\sum w_i U_i$	Baptiste et al. (2004) [2]
$Pm\|p_i = p; size_i\|\sum w_i U_i$	Brucker et al. (2000) [7]
$P\|r_i; p_i = p\|\sum T_i$	Brucker & Kravchenko (2005) [8]
$Pm\|p_i = p; size_i\|\sum T_i$	Brucker et al. (2000) [7]
$P\|p_i = p\|\sum w_i U_i, \sum w_i T_i$	Assignment-problem
$P\|r_i; p_i = 1\|\sum w_i U_i, \sum w_i T_i$	Networkflowproblem

- **maximal pseudopolynomially solvable:**

$P3\|size_i\|C_{max}$	Du & Leung (1989) [15]
$Pm\|r_i\|C_{max}$, $Pm\|\|\sum w_i C_i$, $Pm\|\|\sum w_i U_i$	Lawler et al. (1989) [20]

- **minimal NP-hard:**

	$P2\|\|C_{max}$	Lenstra et al. (1977) [24]
*	$P\|\|C_{max}$	Garey & Johnson (1978) [18]
*	$P\|intree; r_i; p_i = 1\|C_{max}$	Brucker et al. (1977) [5]
*	$P\|p_i = 1; size_i\|C_{max}$	Lloyd (1981) [25]
*	$P\|prec; p_i = 1\|C_{max}$	Ullman (1975) [28]
*	$P2\|chains\|C_{max}$	Du et al. (1991) [16]
*	$P2\|chains; r_i; p_i = 1; size_i\|C_{max}$	Brucker et al. (2000) [7]
*	$P2\|r_i; size_i\|C_{max}$	Lee & Cai (1999) [21]
*	$P3\|chains; p_i = 1; size_i\|C_{max}$	Blazewicz & Liu (1996) [4]
*	$P5\|size_i\|C_{max}$	Du & Leung (1989) [15]
*	$P\|outtree; p_i = 1\|L_{max}$	Brucker et al. (1977) [5]
*	$P2\|chains; p_i = 1; size_i\|L_{max}$	Brucker et al. (2000) [7]
*	$P2\|size_i\|L_{max}$	Lee & Cai (1999) [21]
	$P2\|size_i\|\sum C_i$	Lee & Cai (1999) [21]
*	$P\|intree; r_i; p_i = 1\|\sum C_i$	Lenstra (-) [22]
*	$P\|p_i = 1; size_i\|\sum C_i$	Drozdowski & Dell' Olmo (2000) [14]
*	$P\|prec; p_i = 1\|\sum C_i$	Lenstra & Rinnooy Kan (1978) [23]
*	$P2\|chains\|\sum C_i$	Du et al. (1991) [16]
*	$P2\|intree; p_i = 1; size_i\|\sum C_i$	Zinder & Do (2005) [29]
*	$P2\|outtree; p_i = 1; size_i\|\sum C_i$	Zinder et al. (2005) [30]
*	$P2\|r_i\|\sum C_i$	Single-machine problem
	$P2\|\|\sum w_i C_i$	Bruno et al. (1974) [10]
*	$P\|\|\sum w_i C_i$	Lenstra (-) [22]
*	$P2\|chains; p_i = 1\|\sum w_i C_i$	Timkovsky (1998) [27]
*	$P2\|size_i\|\sum w_i C_i$	Lee & Cai (1999) [21]
*	$P2\|chains; p_i = 1\|\sum U_i, \sum T_i$	Single-machine problem

- **minimal open:**

$P2|r_i; p_i = p; size_i|L_{max}$ $P2|r_i; p_i = 1; size_i|\sum w_i C_i$ $P2|r_i; p_i = 1; size_i|\sum T_i$
$Pm|r_i; p_i = 1; size_i|L_{max}$ $P2|r_i; p_i = 1; size_i|\sum U_i$ $P2|p_i = 1; size_i|\sum w_i T_i$
$P2|chains; p_i = 1; size_i|\sum C_i$

- **maximal open:**

$Pm|chains; r_i; p_i = p; size_i|\sum C_i$ $Pm|r_i; p_i = p; size_i|\sum w_i U_i$ $Pm|r_i; p_i = p; size_i|\sum w_i T_i$

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