$$g(t_p) = d_{\text{opt}}(C_p, C^p(t_p)) \text{ with } C^p(t_p) = S[t_p \dots t_p + c_p - 1] \tag{3.39}$$

with a distance function d. The main point is that we discard any interactions between different core segments.

3.11.2 Parameterization and Conditioning

Instead of looking at the whole string S of length n, we consider prefixes $S[1 \dots j]$. Moreover, instead of considering the whole string T we take prefix $T\langle i \rangle = L_0, C_1, L_1, C_2, \dots, C_i, L_i$ with loop-core segmentation up to C_i, L_i. We compute the following terms $F(i,j)$ that are defined as the minimum value

$$f(t_1, \dots, t_i) = \sum_{p=1}^{i} g(t_p). \tag{3.40}$$

taken over all threadings $t_1, \dots, t_i$ of $S[1 \dots j]$ into the loop-core structure of $T\langle i \rangle$ up to loop segment L_i.

3.11.3 Bellman Principle

... is obviously fulfilled.

3.11.4 Recursive Solution

We start with the case $i = 0$ and $j \in [\lambda_0 \dots \Lambda_0]$. This means that only loop segment L_0 is available for mapping $S[1 \dots j]$. Since $j \in [\lambda_0 \dots \Lambda_0]$ was assumed mapping can be done. Costs are zero, as nothing is summed up. Thus $F(0,j) = 0$.

Next we treat the case $i = 0$ and $j \notin [\lambda_0 \dots \Lambda_0]$. Here no threading exists that fulfils the length constraints. We express this by defining $F(0,j) = \infty$.

Now we treat the case $i > 0$ and $1 \leq j - c_i - \lambda_i + 1$. Whatever we choose as start position t_i for core segment of length c_i in $S[1 \dots j]$, there must be at least $c_i + \lambda_i$ characters between positions t_i and j available for a realization of i^{th} core segment of length c_i and i^{th} loop segment of length at least λ_i. This constrains t_i to $t_i \leq \lambda(i,j) = j - c_i - \lambda_i + 1$. As $1 \leq j - c_i - \lambda_i + 1$ was assumed there is at least one choice for t_i possible. As conversely loop segment numbered i must have length at most Λ_i this constrains t_i to $t_i \geq \Lambda(i,j) = j - c_i - \Lambda_i + 1$. This leads to the following formula.

$$F(i,j) = \min_{\Lambda(i,j) \leq t_i \leq \lambda(i,j)} g(t_i) + F(i-1, t_i - 1) \tag{3.41}$$

Finally we treat the case $i > 0$ and $1 > j - c_i - \lambda_i + 1$. Here, no admissible threadings are possible, thus $F(i,j) = +\infty$.