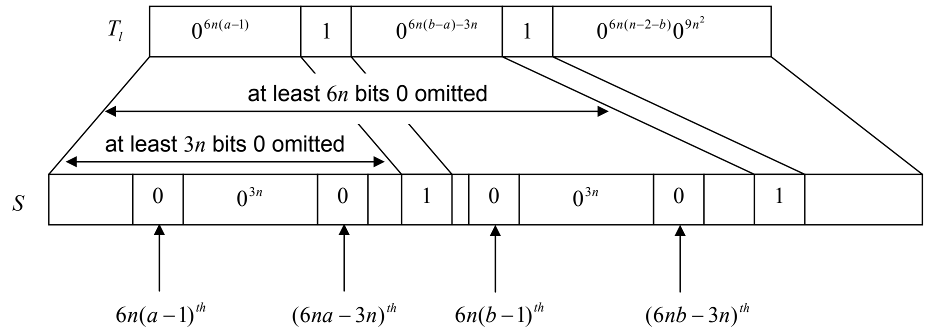

Fig. 5.17. Mapping the two bits 1 of T_l to bits 1 of S

immediately right of the $(6nb-9n)^{\text{th}}$ bit 0. Since already $3n$ bits 0 of S have been omitted and right of the $6n(b-1)^{\text{th}}$ bit 0 of S' there are further $3n$ bits 0, we know that again at least $3n$ bits 0 of S' are omitted when embedding T_l into S. Since T_l has $15n^2+3n$ bits 0, S' has $6n^2+3n$ bits 0, and $6n$ bits 0 of S are not used when embedding T_l into S, we know that the remaining suffix of T_l must be embedded into S''. Thus S'' contains at least $6n+9n^2$ bits 0. Denote the exact number of bits 0 in S'' by $p+9n^2$ with some number $p \geq 6n$. Since each of the strings A_i and S' contain the same number $6n^2+3n$ of bits 0 we know that all of the strings B_j must be embedded into S''. By Lemma 5.20 we know that S'' must contain at least $(9n^2+1)/(p+1)$ many bits 1. Thus S'' has length at least $p+9n^2+(9n^2+1)/(p+1)$. Since $K = 15n^2+10n+k$ was the length of S and S' contained $6n^2+3n$ bits 0 and at least n bits 1, we conversely know that the length of S'' can be at most $9n^2+6n+k$. This results in the following inequality which we lead to a contradiction (using $k \leq n$) by a sequence of transformation steps:

$$p+9n^2+\frac{9n^2+1}{p+1} \leq 9n^2+6n+k \leq 9n^2+7n$$

$$\Leftrightarrow p+\frac{9n^2+1}{p+1} \leq 7n\ . \qquad (i)$$

Now we show

$$6n+\frac{9n^2+1}{6n+1} \leq p+\frac{9n^2+1}{p+1} \qquad (ii)$$

by the following equivalence transformations.